Question 1: Battery manufacturing problem.

Optimal expected total return = 4.8

Optimal policy:

Carry out the research

If it is completely successful, manufacture the battery

If it is partially successful, do not manufacture the battery

Question 2: Wage negotiation problem.

Minimum expected total cost = 10.62

Optimal policy:

Make an intermediate offer

Question 3: Car manufacturing problem.

(a) Optimal expected total return = 28.4

Optimal policy:

Choose high capacity

(b) Optimal expected total return = 28.4

Optimal policy:

Do not undertake the survey

Question 4: Pile ordering problem.

(a) Optimal expected loss = £1100

Optimal policy:

Order 12m piles for both north and south piers

(b) Optimal expected loss = £920

Optimal policy:

Order 50 piles @ 11m and 50 piles @ 12m

Question 5: Plant extension problem.

(a) Optimal expected annual profit increase = 1.493

Optimal policy:

Order now & choose capacity 13

(b) Maximum expected utility = 0.8 (£1.4m)

Optimal policy:

Observe d_1. If d_1 = 10, choose capacity 12

If d_1 = 11, choose capacity 13

BATTERY MANUFACTURING PROBLEM

• Probem Formulation

ACTIONS:

$K=1$ do research $\Rightarrow \begin{cases} \text{manufacture} \\ \text{don't manufacture} \end{cases}$

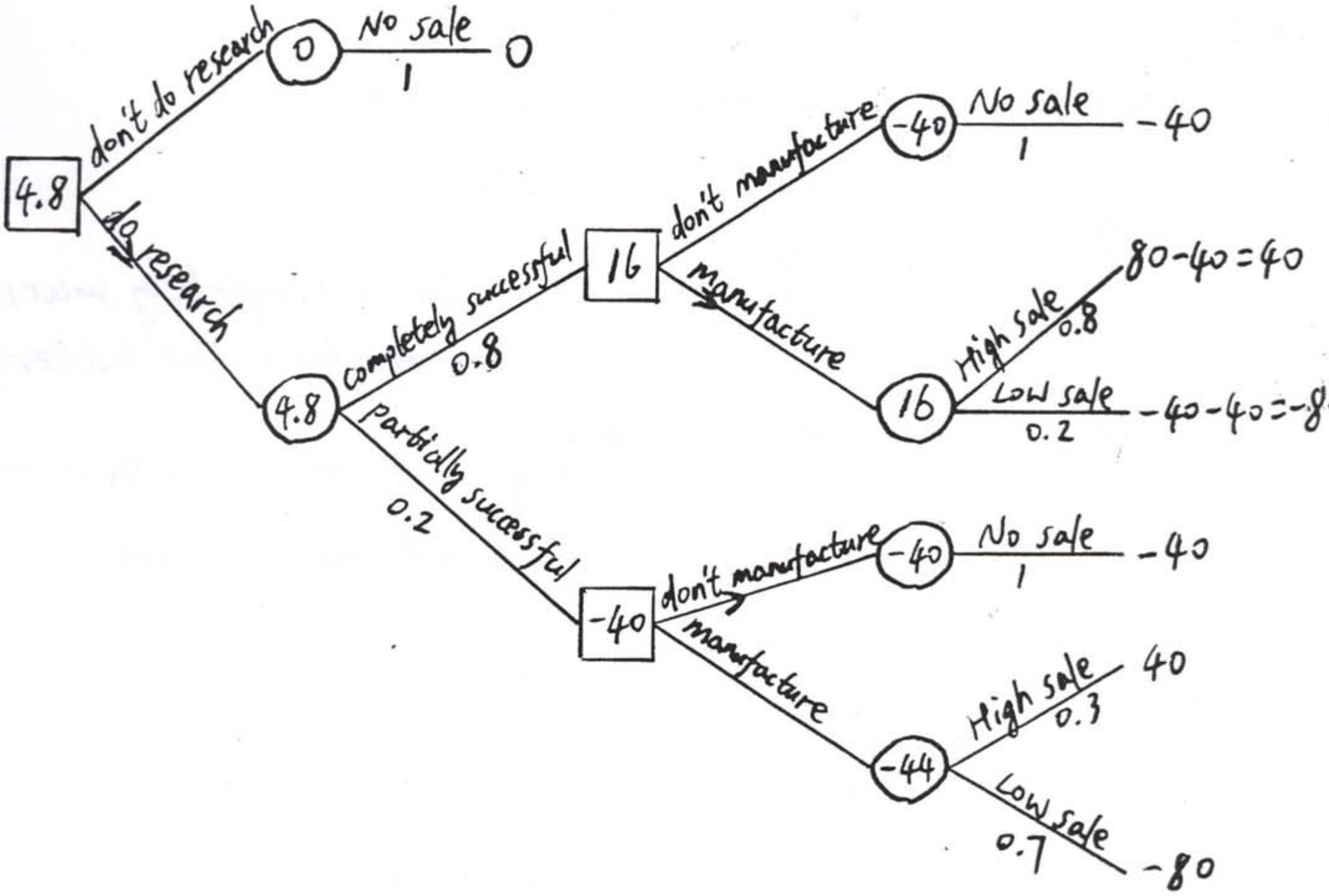
$K=2$ don't do research

OUTCOME:

$O=1$ No sale

$O=2$ Low Sale

$O=3$ High sale



• Numerical Solution

i Completely successful & manufacture the battery

$$EMV = 0.8 \times 40 + 0.2 \times (-80) = 16$$

ii. Partially successful & manufacture the battery

$$EMV = 0.3 \times 40 + 0.7 \times (-80) = -44$$

iii. Do research

$$EMV = 0.8 \times 16 + 0.2 \times (-40) = 4.8$$

• Answer

i. Maximum Expected Return: 4.8

ii. Optimal Policy

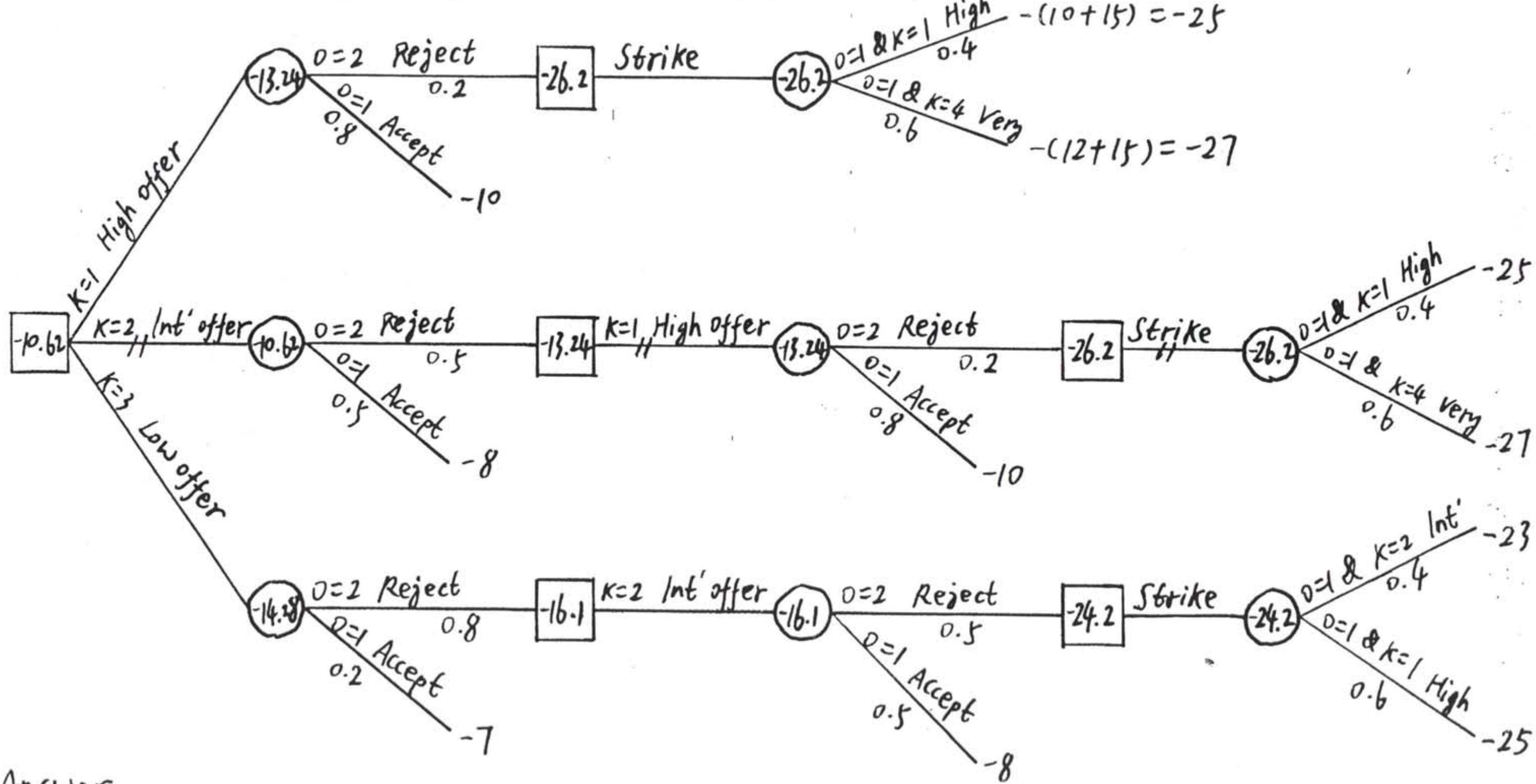
Carry out the research
- If it is completely successful, manufacture the battery
- If it is partially successful do not manufacture the batte

WAGE NEGOTIATION PROBLEM

- Problem Formulation

ACTION: K=1 high offer, K=2 Intermediate offer, K=3 low offer, K=4 very high offer

OUTCOME: O=1 accept offer, O=2 reject offer



- Answer

i. Minimum Expected Cost: 10.62 ; ii Optimal policy: make an intermediate offer.

CAR MANUFACTURING PROBLEM

(a) Problem Formulation

ACTION: $K=1$ choose high capacity

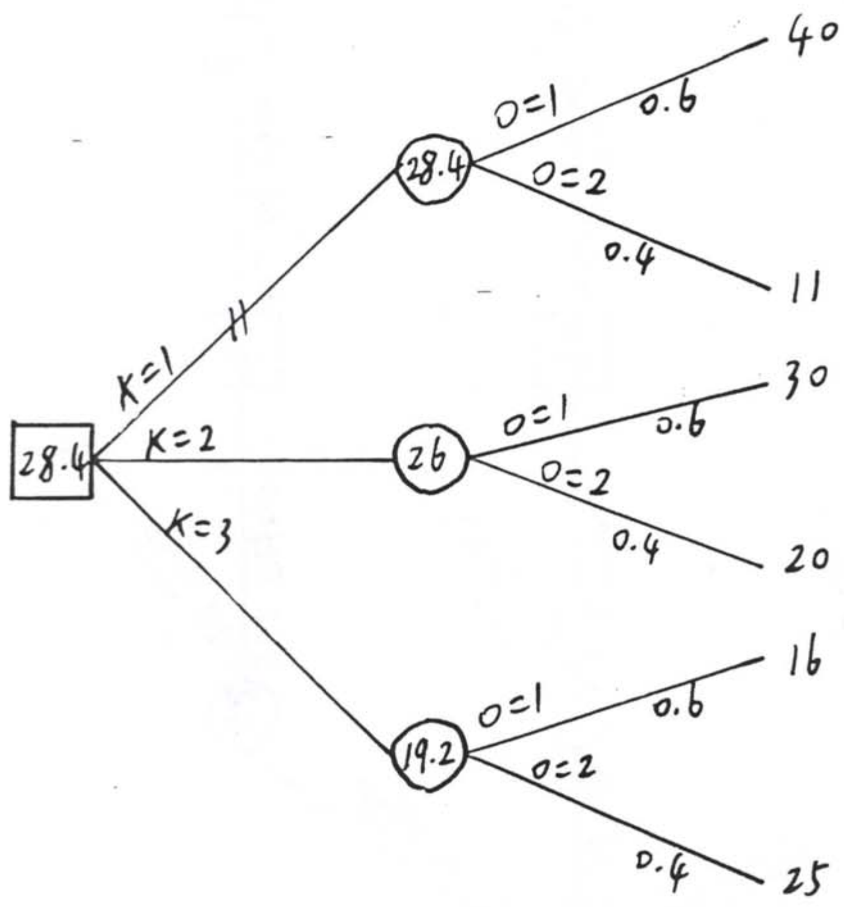
$K=2$ choose medium capacity

$K=3$ choose low capacity

OUTCOME: $O=1$ high demand

$O=2$ low demand

Expected value: maximum expected return



• Answer.

i. Choose high capacity

ii. Optimal expected Return = 28.4

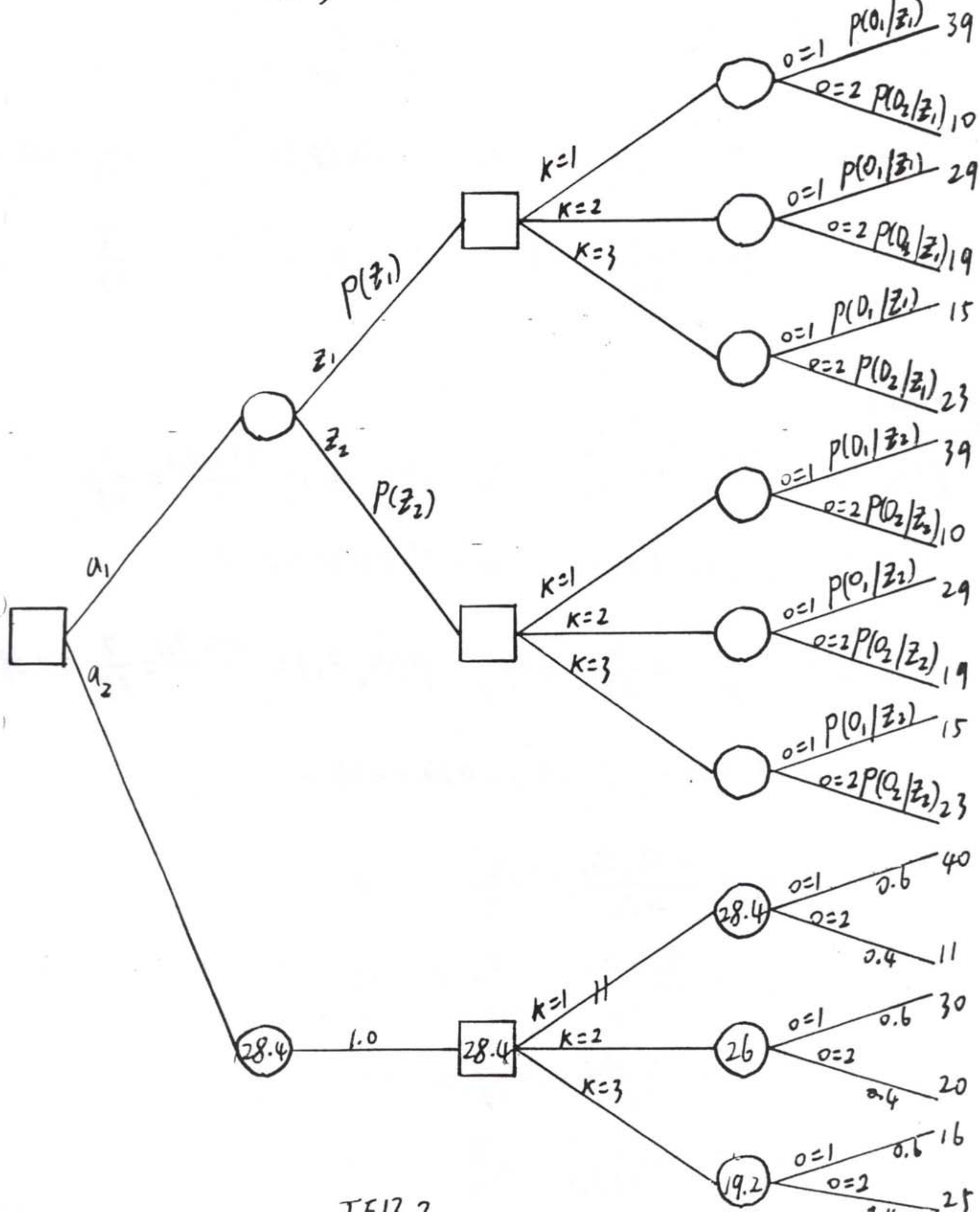
(b) Problem Formulation

ACTION: a_1 undertake survey $\Rightarrow k=1, 2, 3$

a_2 don't undertake survey $\Rightarrow k=1, 2, 3$

OUTCOME: z_1 forecast 1 $\Rightarrow o=1, 2$

z_2 forecast 2 $\Rightarrow o=1, 2$



• Calculation of Probabilities

Approach 1:

	O_1	O_2		
Z_1	9	3	12	$N(Z_1)=12$
Z_2	6	7	13	$N(Z_2)=13$
			25	$N=25$

$N(O_1, Z_1)=9,\quad N(O_2, Z_1)=3,\quad N(O_1, Z_2)=6,\quad N(O_2, Z_2)=7$

$$P(Z_1)=\frac{N(Z_1)}{N}=\frac{12}{25}=0.48,\quad P(Z_2)=\frac{N(Z_2)}{N}=\frac{13}{25}=0.52$$

$$P(O_1/Z_1)=\frac{N(O_1, Z_1)}{N(Z_1)}=\frac{9}{12}=0.75,\quad P(O_2/Z_1)=\frac{N(O_2, Z_1)}{N(Z_1)}=\frac{3}{12}=0.25$$

$$P(O_1/Z_2)=\frac{N(O_1, Z_2)}{N(Z_2)}=\frac{6}{13}=0.46,\quad P(O_2/Z_2)=\frac{N(O_2, Z_2)}{N(Z_2)}=\frac{7}{13}=0.54$$

Approach 2:

$$P(O_1, Z_1)=\frac{N(O_1, Z_1)}{N}=\frac{9}{25}=0.36,\quad P(O_2, Z_1)=\frac{N(O_2, Z_1)}{N}=\frac{3}{25}=0.12$$

$\therefore\ P(Z_1)=P(O_1, Z_1)+P(O_2, Z_1)=0.36+0.12=0.48$

$$P(O_1, Z_2)=\frac{N(O_1, Z_2)}{N}=\frac{6}{25}=0.24,\quad P(O_2, Z_2)=\frac{N(O_2, Z_2)}{N}=\frac{7}{25}=0.28$$

$\therefore\ P(Z_2)=P(O_1, Z_2)+P(O_2, Z_2)=0.24+0.28=0.52$

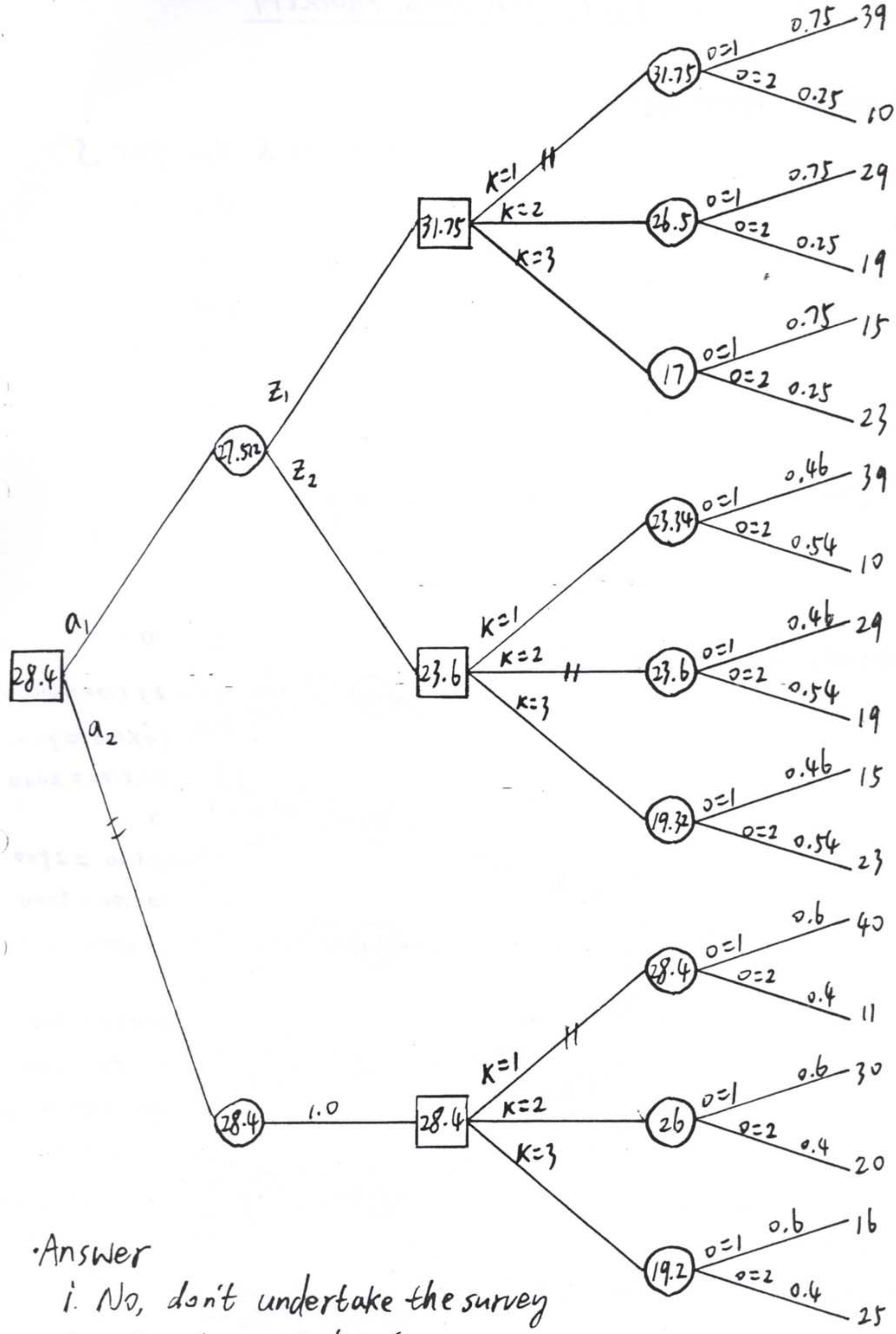
$$P(O_1/Z_1)=\frac{P(O_1, Z_1)}{P(Z_1)}=\frac{0.36}{0.48}=0.75$$

$$P(O_2/Z_1)=\frac{P(O_2, Z_1)}{P(Z_1)}=\frac{0.12}{0.48}=0.25$$

$$P(O_1/Z_2)=\frac{P(O_1, Z_2)}{P(Z_2)}=\frac{0.24}{0.52}=0.46$$

$$P(O_2/Z_2)=\frac{P(O_2, Z_2)}{P(Z_2)}=\frac{0.28}{0.52}=0.54$$

• Numerical Solution



·Answer

i. No, don't undertake the survey

ii Optimal expected return = 28.4

PILE ORDERING PROBLEM

- Problem Formulation

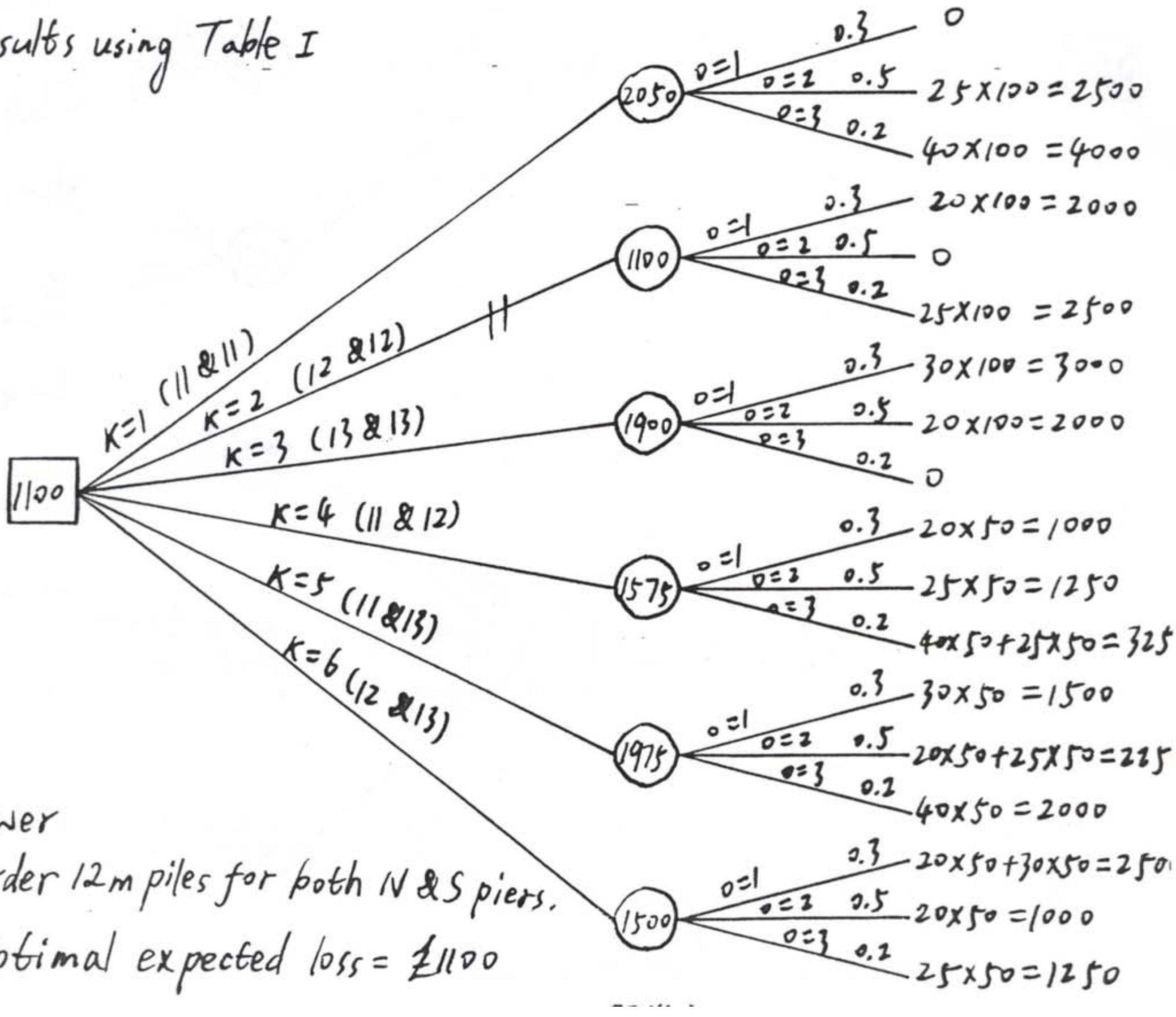
ACTIONS:

K=1	order	11 m	piles for N &	11m	for S
2	″	12	″	12m	″
3	″	13	″	13m	″
4	″	11	″	12m	″
5	″	11	″	13m	″
6	″	12	″	13m	″

OUTCOMES:

O=1	rock is	11 m	deep
O=2	″	12 m	″
O=3	″	13 m	″

(a) Results using Table I



- Answer

i. Order 12m piles for both N & S piers.

ii. Optimal expected loss = £1100

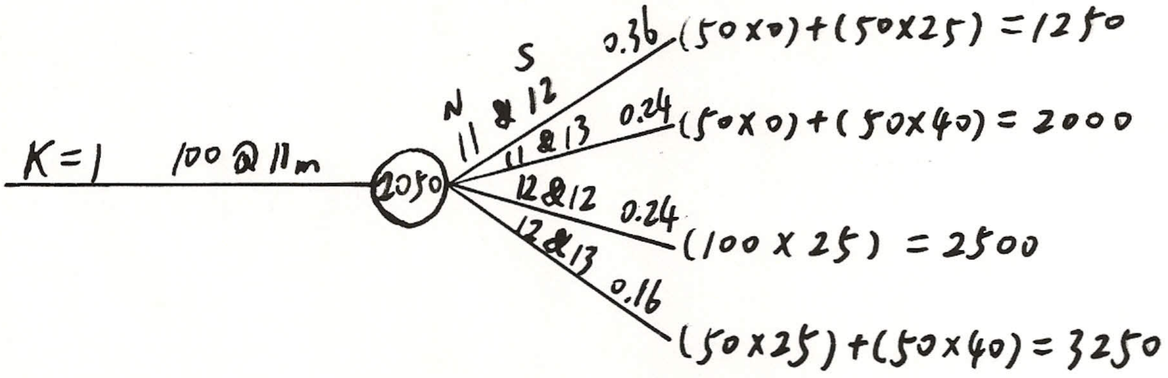
(b) Results using Table I

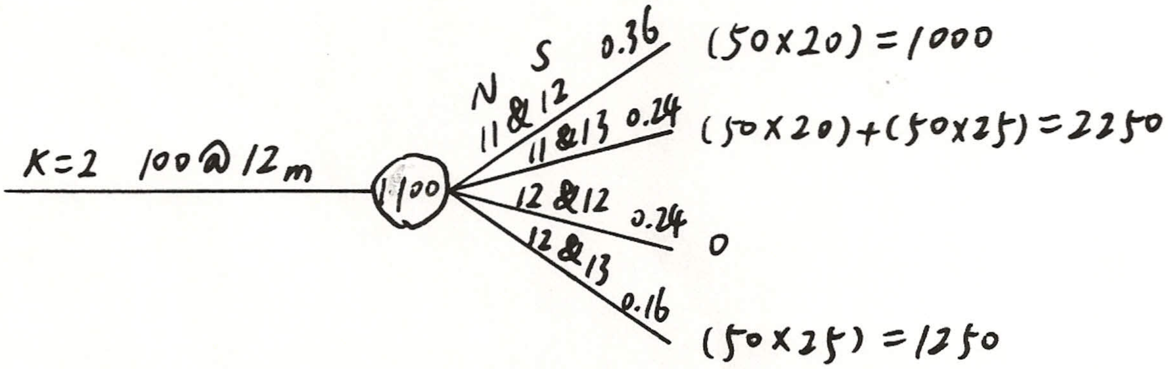
ACTION:

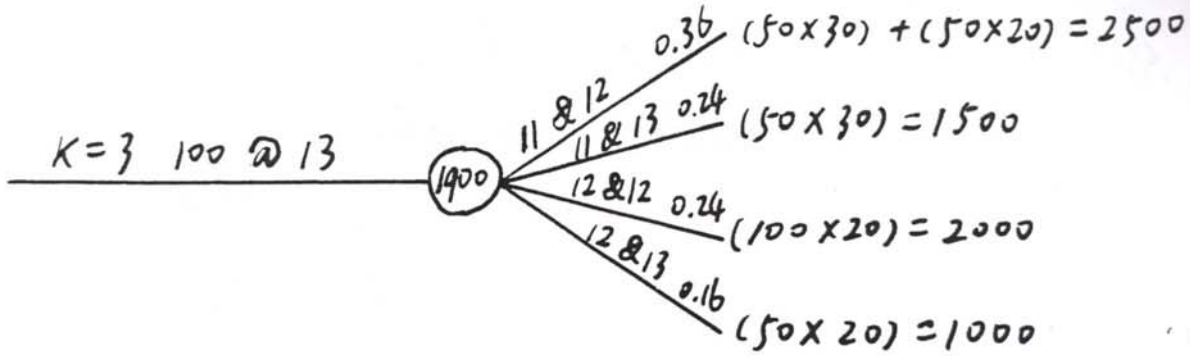
K = 1	Order	100	piles at	11 m
2	order	100	piles at	12 m
3	"	100	"	13 m
4	"	50	"	11 m & 50 at 12 m
5	"	50	"	11 m & 50 at 13 m
6	"	50	"	12 m & 50 at 13 m

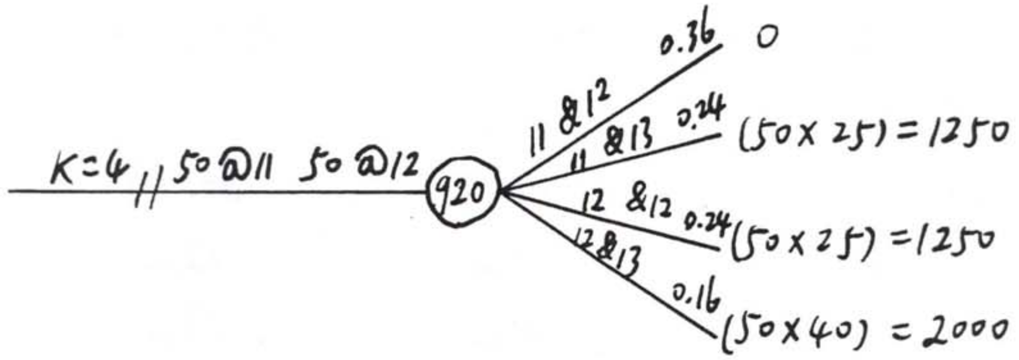
OUTCOMES:

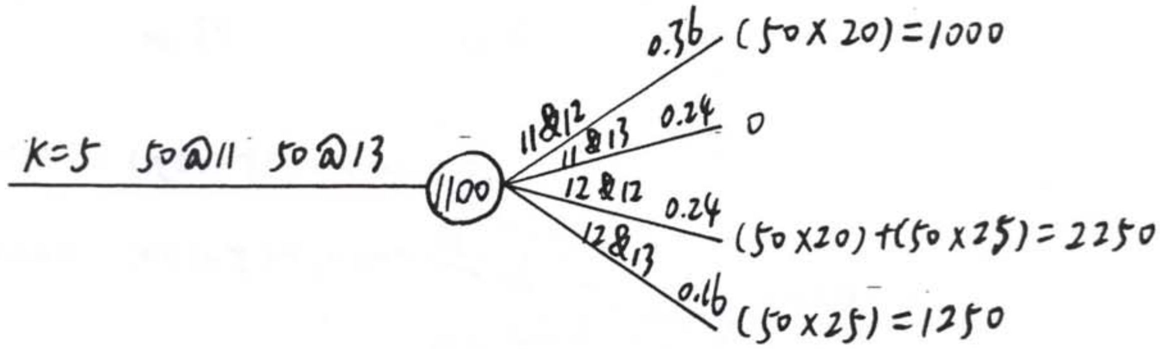
	North	South
O = 1	11 m	12 m
2	11 m	13 m
3	12 m	12 m
4	12 m	13 m

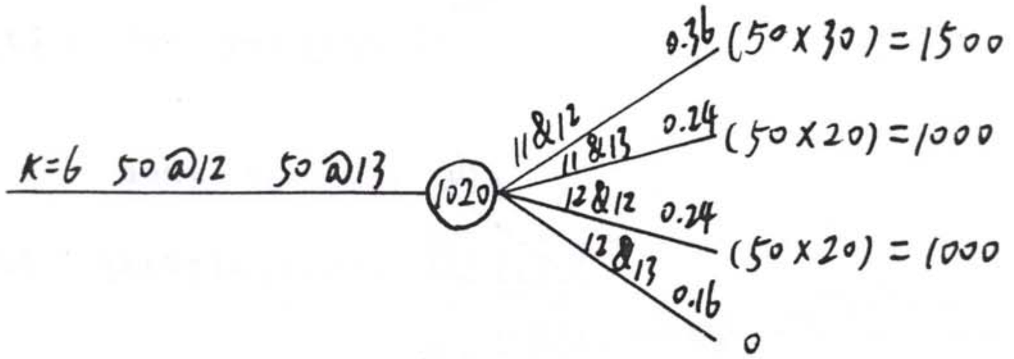












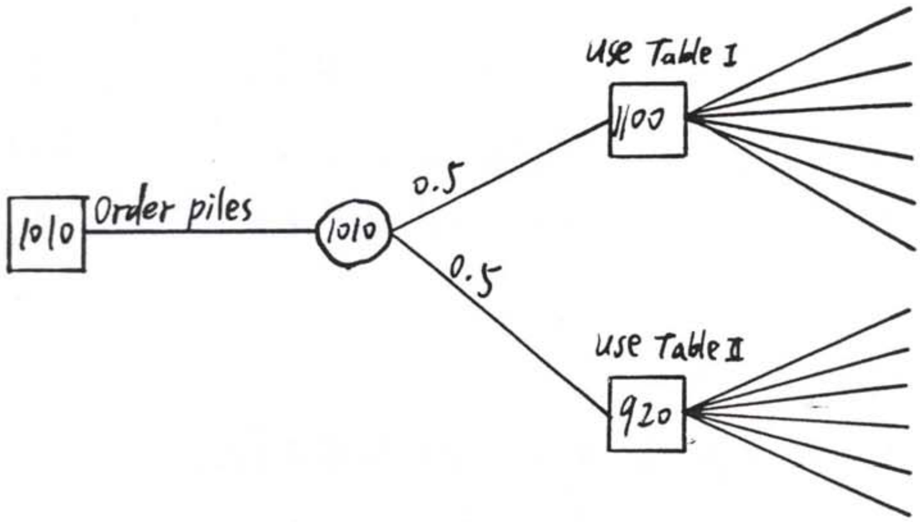
- Answer

i. Order 50 piles @ 11m and 50 piles @ 12 m

ii Expected loss = £920

- Results using both Table I and Table II

If table I and table II are equally probable, the problem can be formulated as follows



- Answer

Expected loss is £1010

PLANT EXTENSION PROBLEM

- Problem Formulation

ACTIONS: $K=1$ observe d_1 ~~then~~ order | order 12 capacity
$K=2$ order now | order 13 capacity
| order 14 capacity

OUTCOME: $d_1 = 10$ | Demand will be 12 ($d_3=12$)
$d_1 = 11$ | Demand will be 13 ($d_3=13$)
| Demand will be 14 ($d_3=14$)

- Calculation of Probabilities

$$P(d_3=j\,,\ d_1=i) = p(d_3=j/d_1=i)\,p(d_1=i)$$

$$P(d_3=j/d_1=i) = \begin{array}{c c} & \begin{array}{ccc} (j=)\ 12 & 13 & 14 \end{array} \\ \begin{array}{c} (i=)\ 10 \\ 11 \end{array} & \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.14 & 0.46 & 0.4 \end{bmatrix} \end{array}$$

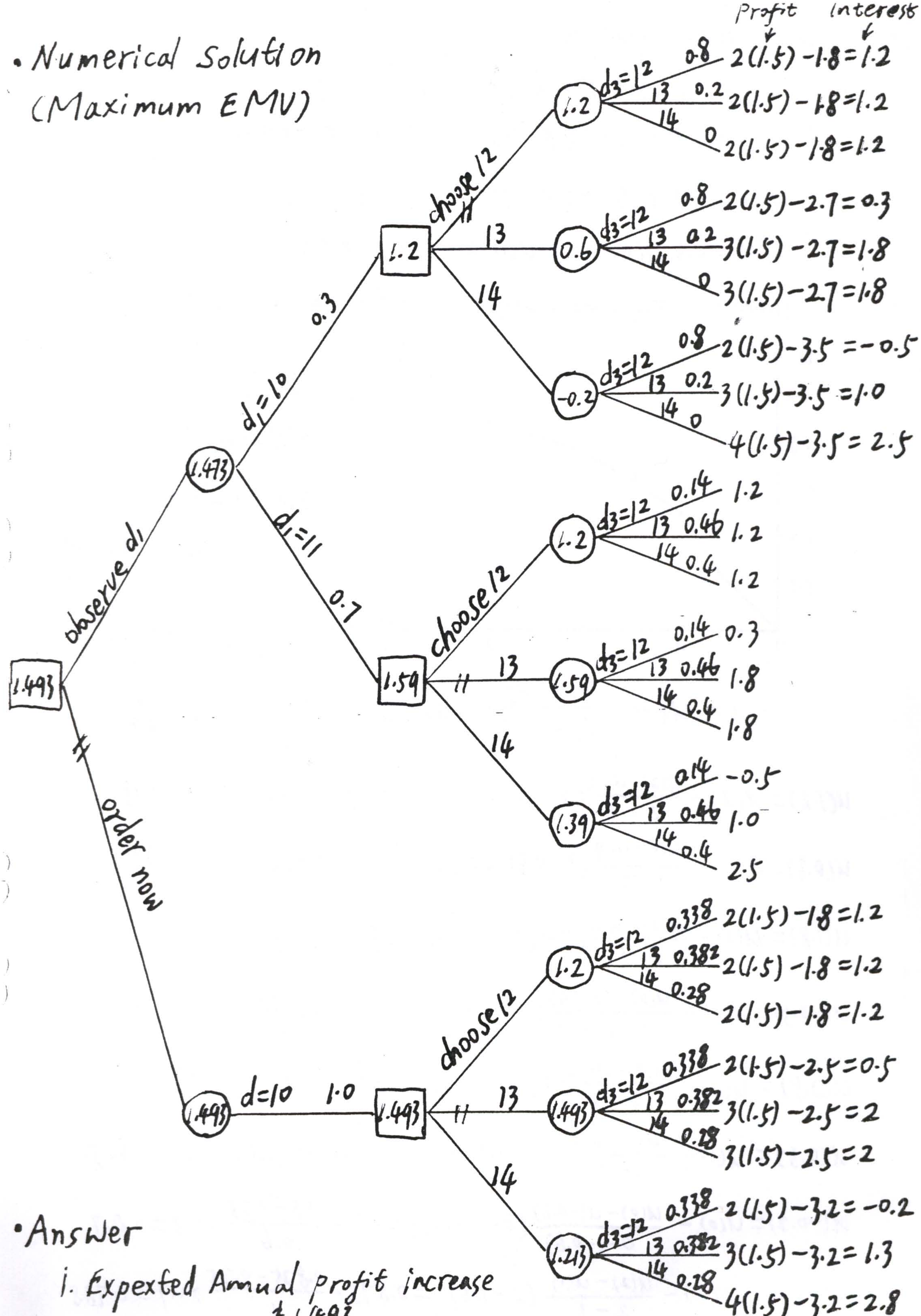
$$P(d_1=i) \qquad \begin{array}{c c} \begin{array}{c} i=10 \\ 11 \end{array} & \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix} \end{array}$$

$$\therefore P(d_3=j\,,\ d_1=i) = \begin{array}{c c} & \begin{array}{ccc} (j=)\ 12 & 13 & 14 \end{array} \\ \begin{array}{c} (i=)10 \\ 11 \end{array} & \begin{bmatrix} 0.24 & 0.06 & 0 \\ 0.098 & 0.322 & 0.28 \end{bmatrix} \end{array}$$

$$P(d_3=j) = P(d_3=j\,,\ d_1=10) + p(d_3=j\,,\ d_1=11)$$

$$P(d_3=j) = \begin{array}{c} \begin{array}{ccc} j=12 & 13 & 14 \end{array} \\ \begin{bmatrix} 0.24 & 0.06 & 0 \\ + & + & + \\ 0.098 & 0.322 & 0.28 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 0.338 & 0.382 & 0.28 \end{bmatrix} \end{array}$$

- Numerical Solution (Maximum EMV)



- Answer

  i. Expexted Annual profit increase £1.493

  ii. Optimal policy: Order now & choose capacity 13

- Estimation of Utilities

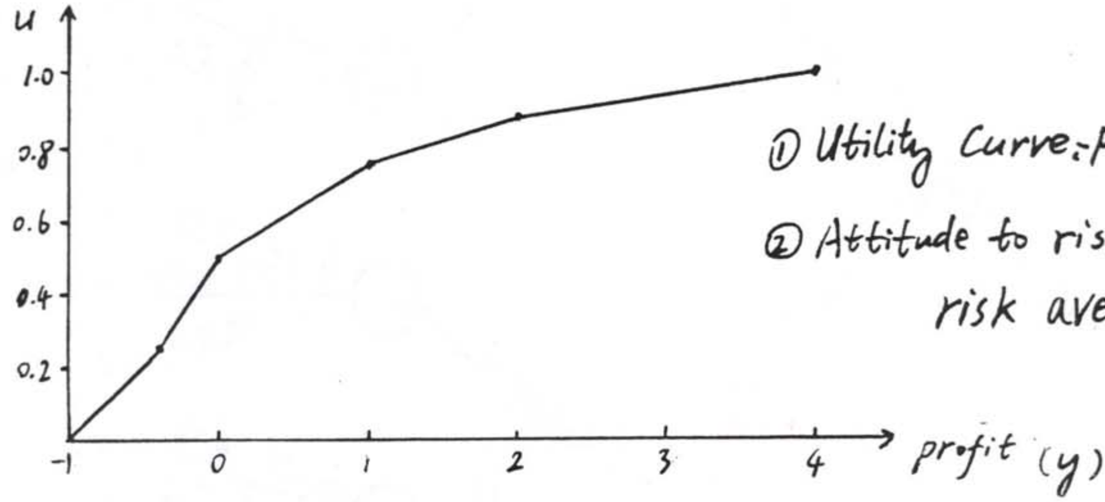
Let $U(4)=1$, $U(-1)=0$

$\therefore \quad U(0) = 0.5\,U(4) + 0.5\,U(-1) = 0.5\times 1 + 0.5\times 0 = 0.5$

$U(1) = 0.5\,U(4) + 0.5\,U(0) = 0.5\times 1 + 0.5\times 0.5 = 0.75$

$U(2) = 0.5\,U(4) + 0.5\,U(1) = 0.5\times 1 + 0.5\times 0.75 = 0.875$

$U(-0.6) = 0.5\,U(0) + 0.5\,U(-1) = 0.5\times 0.5 + 0.5\times 0 = 0.25$



① Utility Curve: Piecewise line

② Attitude to risk:
risk averse

$$U = U_i - \frac{U_i - U_j}{y_i - y_j}(y_i - y) \quad \text{if } y_j \le y \le y_i$$

$U(1.2) = U(2) - \frac{U(2)-U(1)}{2-1}(2-1.2) = 0.875 - (0.875-0.75)\times 0.8 = 0.775$

$U(0.3) = U(1) - \frac{U(1)-U(0)}{1-0}(1-0.3) = 0.75 - (0.75-0.5)\times 0.7 = 0.575$

$U(1.8) = U(2) - \frac{U(2)-U(1)}{2-1}(2-1.8) = 0.875 - (0.875-0.75)\times 0.2 = 0.85$

$U(-0.5) = U(0) - \frac{U(0)-U(-0.6)}{0-(-0.6)}(0-(-0.5)) = 0.5 - \frac{0.5-0.25}{0.6}\times 0.5 = 0.292$

$U(2.5) = U(4) - \frac{U(4)-U(2)}{4-2}(4-2.5) = 1 - \frac{1}{2}(1-0.875)\times 1.5 = 0.906$

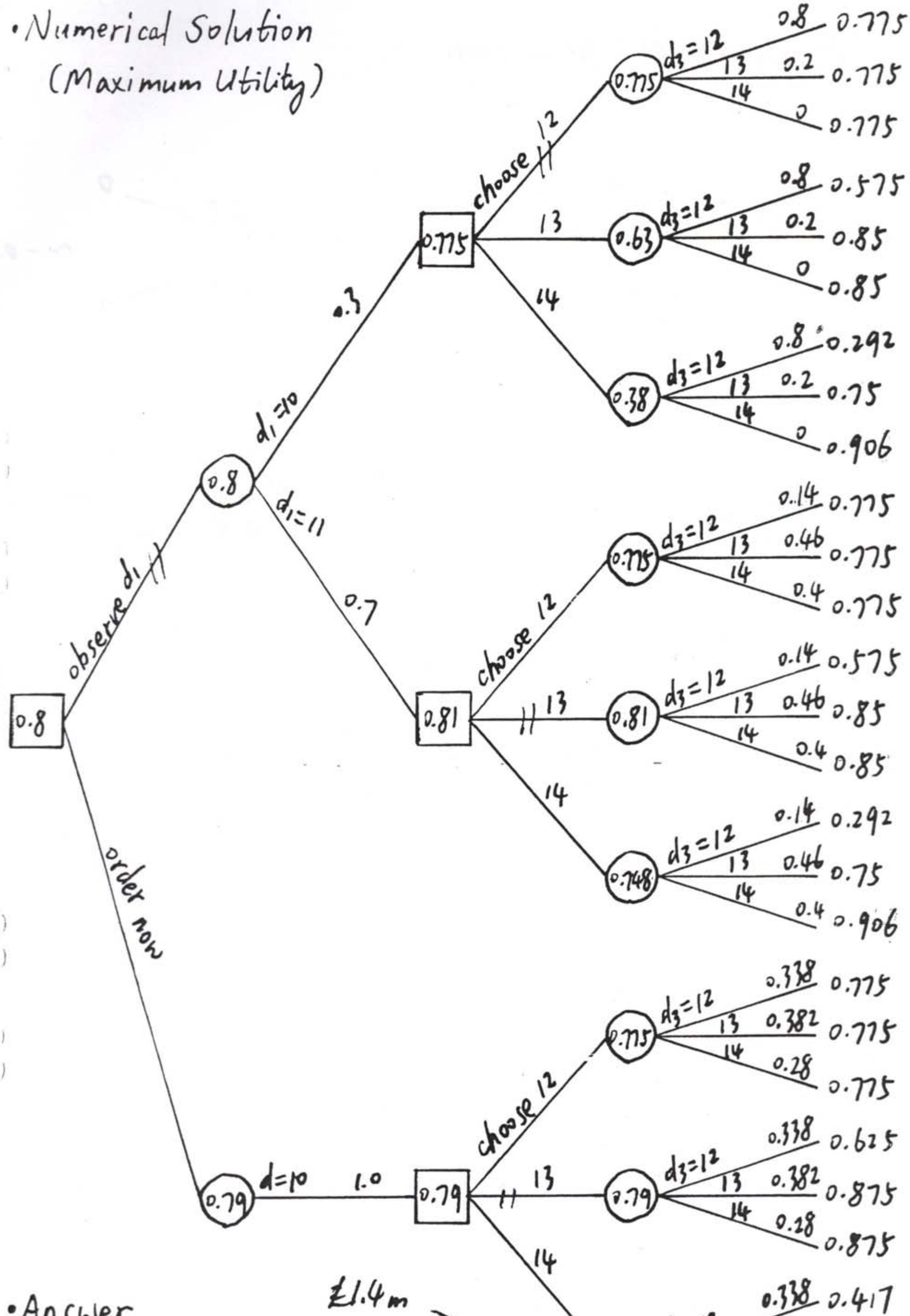
$U(0.5) = U(1) - \frac{U(1)-U(0)}{1-0}(1-0.5) = 0.75 - (0.75-0.5)\times 0.5 = 0.625$

$U(-0.2) = U(0) - \frac{U(0)-U(-0.6)}{0-(-0.6)}(0-(-0.2)) = 0.5 - \frac{0.5-0.25}{0.6}\times 0.2 = 0.417$

$U(1.3) = U(2) - \frac{U(2)-U(1)}{2-1}(2-1.3) = 0.875 - \frac{0.875-0.75}{1}\times 0.7 = 0.788$

$U(2.8) = U(4) - \frac{U(4)-U(2)}{4-2}(4-2.8) = 1 - \frac{1}{2}(1-0.875)\times 1.2 = 0.925$

• Numerical Solution
(Maximum Utility)



• Answer

i. Maximum Expected Utility = 0.8 ← £1.4m

ii. Optimal Policy:

Observe d_1. If $d_1=10$, choose capcity 12; if $d_1=11$, choose 13