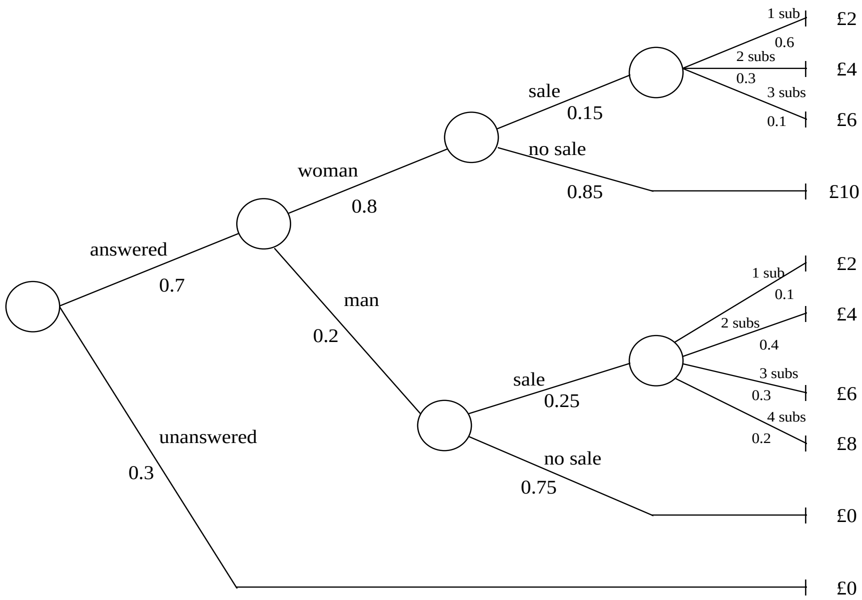
SIMULATION EXERCISE SOLUTIONS

1. The logic of the situation is perhaps best represented here by a decision tree:



From the tree it is clear there are 6 sets of chance outcomes. We need to determine which (uniform) random numbers (from tables) will signal the occurrence of each outcome:

Outcome Answered	Rel Freq	Cum Freq	Random Numbers
Yes	0.7	0.7	0 - 69
No	0.3	1.0	70 - 99

Outcome Answer by	Rel Freq	Cum Freq	Random Numbers
Woman	0.8	0.8	0 - 79
Man	0.2	1.0	80 - 99

Outcome Sale to Woman	Rel Freq	Cum Freq	Random Numbers
Yes	0.15	0.15	0 - 14
No	0.85	1.00	15 - 99

Outcome Sale to Man	Rel Freq	Cum Freq	Random Numbers
Yes	0.25	0.25	0 - 24
No	0.75	1.00	25 - 99

Outcome Subscripts to Woman	Rel Freq	Cum Freq	Random Numbers
1	0.60	0.60	0 - 59
2	0.30	0.90	60 - 89
3	0.10	1.00	90 - 99

Outcome Subscripts to Man	Rel Freq	Cum Freq	Random Numbers
1	0.10	0.10	0 - 9
2	0.40	0.50	10 - 49
3	0.30	0.80	50 - 79
4	0.20	1.00	80 - 99

a) simulation of 25 calls:

Call	**Answered**		**Woman/Man**		**Woman Sale**		**Woman Subs**		**Man Sale**		**Man Subs**		**Sales**	**Subscriptions**
	RN	*result*	*RN*	*result*	*RN*	*result*	*RN*	*result*	*RN*	*result*	*RN*	*result*		
1	96	No											0	0
2	64	Yes	53	Woman	5	Yes	91	3					1	3
3	3	Yes	61	Woman	4	Yes	21	1					1	1
4	36	Yes	84	Man					63	No			0	0
5	67	Yes	23	Woman	49	No							0	0
6	90	No											0	0
7	11	Yes	18	Woman	91	No							0	0
8	92	No											0	0
9	32	Yes	76	Woman	39	No							0	0
10	72	No											0	0
11	15	Yes	56	Woman	24	No							0	0
12	40	Yes	37	Woman	13	Yes	79	2					1	2
13	99	No											0	0
14	82	No											0	0
15	3	Yes	35	Woman	58	No							0	0
16	31	Yes	82	Man					22	Yes	22	2	1	2
17	64	Yes	8	Woman	72	No							0	0
18	98	No											0	0
19	64	Yes	35	Woman	16	No							0	0
20	91	No											0	0
21	11	Yes	19	Woman	85	No							0	0
22	8	Yes	85	Man					20	Yes	12	2	1	2
23	53	Yes	70	Woman	90	No							0	0
24	25	Yes	37	Woman	46	No							0	0
25	87	No											0	0

b) Total sales = 5
Total subscriptions = 10
Total profit = £20

c) From the decision tree, working from right to left:

EV node 4 = $0.6 \times 2 + 0.3 \times 4 + 0.1 \times 6$ = £3.00

EV node 3 = $0.15 \times 3 + 0.85 \times 0$ = £0.45

EV node 6 = $0.1 \times 2 + 0.4 \times 4 + 0.3 \times 6 + 0.2 \times 8$ = £5.20

EV node 5 = $0.25 \times 5.2 + 0.75 \times 0$ = £1.30

EV node 2 = $0.8 \times 0.45 + 0.2 \times 1.3$ = £0.62

EV node 1 = $0.7 \times 0.62 + 0.3 \times 0$ = £0.434

Therefore the total expected profit would be 25×0.434 = £10.85

Expected number of sales per call would be $0.7 \times (0.8 \times 0.15 + 0.2 \times 0.25) = 0.119$

Therefore the total expected number of calls would be $25 \times 0.119 = 2.975$

d) The simulation result represents a one off 'snapshot' of 25 particular calls, whereas the expected results are the long run averages - i.e. the average of an infinite number of calls. Despite the fact that our simulation represents a sample of 25 independent calls, we have obtained results considerably larger than the expected values. The results of a stochastic simulation such as this depend on the random numbers used, and here it would appear that the particular random numbers used have produced a sample of unusually fruitful calls.

If doing this 'for real' we would use a computer to generate a large number of runs similar to the manual one above, and take averages and obtain confidence intervals.

NB. This problem can be done more quickly and simply by calculating the one-step probabilities of ending up at each node, and using the random numbers in pairs to provide the four-figure accuracy required in this case.

2. i)

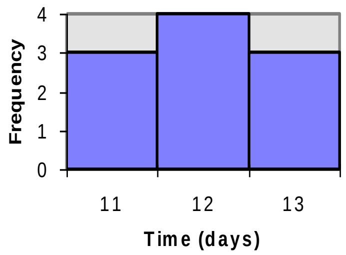
Activity	**Activity Time (Weeks)**	**Probability**	**cum**	**min**	**max**
	1	0.2	0.2	0	19
A	2	0.4	0.6	20	59
	3	0.4	1	60	99
	3	0.2	0.2	0	19
B	4	0.45	0.65	20	64
	5	0.2	0.85	65	84
	6	0.15	1	85	99
	5	0.3	0.3	0	29
C	6	0.3	0.6	30	59
	7	0.3	0.9	60	89
	8	0.1	1	90	99
	4	0.1	0.1	0	9
D	5	0.8	0.9	10	89
	6	0.1	1	90	99

Simulation:

Run	**StartA**	**rndA**	**DurA**	**EndA**	**StartC**	**RndC**	**DurC**	**EndC**	**StartB**	**RndB**	**DurB**	**EndB**	**StartD**	**RndD**	**DurD**	**EndD**	**(PATH)**
1	0	*96*	3	3	0	*64*	7	7	3	*53*	4	7	7	*5*	4	11	**=**
2	0	*91*	3	3	0	*3*	5	5	3	*61*	4	7	7	*4*	4	11	**B**
3	0	*21*	2	2	0	*36*	6	6	2	*84*	5	7	7	*63*	5	12	**B**
4	0	*67*	3	3	0	*23*	5	5	3	*49*	4	7	7	*90*	6	13	**B**
5	0	*11*	1	1	0	*18*	5	5	1	*91*	6	7	7	*92*	6	13	**B**
6	0	*32*	2	2	0	*76*	7	7	2	*39*	4	6	7	*72*	5	12	**C**
7	0	*15*	1	1	0	*56*	6	6	1	*24*	4	5	6	*40*	5	11	**C**
8	0	*37*	2	2	0	*13*	5	5	2	*79*	5	7	7	*99*	6	13	**B**
9	0	*82*	3	3	0	*3*	5	5	3	*35*	4	7	7	*58*	5	12	**B**
10	0	*31*	2	2	0	*82*	7	7	2	*22*	4	6	7	*22*	5	12	**C**

mean time = 12 weeks

{precise values different if you evaluate B before C} ii)



iii) Analytical CPM models (e.g. PERT) would be quicker, but simulation is necessary here as:

the activity completion times don't fit the Beta distribution assumption or additivity of variance on critical path

the critical path is quite finely balance between two alternatives

Simulation can produce a distribution of outcomes of (mathematically) complex situations as here, which may help with risk analysis

can cope with more complex situations but validation can be difficult

3. Pass or fail efficacy and safety trials : P(success) = 0.7 × 0.9 = 0.63

Random Number	Outcome
0 - 62	**Passes both efficacy and safety trials**
63 - 99	**Fails one or other of trials**

Duration of efficacy and safety trials under actual conditions of use:

Probability	Cumulative probability	Random numbers	Outcome: Period (months)
25%	0.25	**0 - 24**	**9**
25%	0.50	**25 - 49**	**10**
25%	0.75	**50 - 74**	**11**
25%	1.00	**75- 99**	**12**

Duration of drawing up formal application to the licensing authorities:

Probability	Cumulative probability	Random numbers	Outcome: Period (months)
25%	0.25	**0 - 24**	**6**
25%	0.50	**25 - 49**	**7**
25%	0.75	**50 - 74**	**8**
25%	1.00	**75 - 99**	**9**

Duration of wait for licence:

Probability	Cumulative probability	Random numbers	Outcome: Period (months)
5%	0.05	**0 - 4**	**15**
10%	0.15	**5 - 14**	**16**
20%	0.35	**15 - 34**	**17**
30%	0.65	**35 - 64**	**18**
20%	0.85	**65 - 84**	**19**
10%	0.95	**85 - 94**	**20**
5%	1.00	**95 - 99**	**21**

Number of competitors

Probability	Cumulative probability	Random numbers	Outcome: Competitors
20%	0.20	**0 - 19**	**0**
40%	0.60	**20 - 59**	**1**
30%	0.90	**60 - 89**	**2**
10%	1.00	**90 - 99**	**3**

TT = Duration of E&S trials under actual conditions of use
+ Duration of drawing up application to the licensing authorities
+ Duration of wait for licence

Time (months) on market during next five years, TM = 5 × 12 - (1 + TT + 1)
Total Revenue generated over 5 years, TR = TM/12 × £10 million × no. of products on market

i) simulation: (15 runs)

Run	Pass/Fail		E&S trials		written application		wait for licence		Competitors		Total Time	Time on Market	Total Revenue
	R/N	Outcome	R/N	Outcome (months)	R/N	Outcome (months)	R/N	Outcome (months)	R/N	Outcome	TT months	TM months	TR
1	*96*	Fail											£0
2	*64*	Fail											£0
3	*53*	Pass	*5*	9	*91*	9	*3*	15	*61*	2	33	25	£6,944,444
4	*4*	Pass	*21*	9	*36*	7	*84*	19	*63*	2	35	23	£6,388,889
5	*67*	Fail											£0
6	*23*	Pass	*49*	10	*90*	9	*11*	16	*18*	0	35	23	£19,166,667
7	*91*	Fail											£0
8	*92*	Fail											£0
9	*32*	Pass	*76*	12	*39*	7	*72*	19	*15*	0	38	20	£16,666,667
10	*56*	Pass	*24*	9	*40*	7	*37*	18	*13*	0	34	24	£20,000,000
11	*79*	Fail											£0
12	*99*	Fail											£0
13	*82*	Fail											£0
14	*3*	Pass	*35*	10	*58*	8	*31*	17	*82*	2	35	23	£6,388,889
15	*22*	Pass	*22*	9	*64*	8	*8*	16	*72*	2	33	25	£6,944,444

Expected total revenue (mean) = £5,500,000
Standard deviation = £7,413,671

Standard error, $\frac{s}{\sqrt{n}}$ = £1,914,202

95% confidence interval: $\mu \approx \bar{X} \pm 1.96 \frac{s}{\sqrt{n}}$

The sample is not really 'large', but we'll use the above as an approximation (should also really use *t*-distribution)

So a 95% CI is roughly £5,500,000 ± 3,751,836 or (£1,748,164; £9,251,836)

ii) Much larger simulations could be performed for each drug in the R&D process to generated total revenue or profit distributions (probably incorporating NPV calculations). The relative risks involved in each drug could then be compared looking at spreads of outcomes as well as expected values. A focus might be, for example, the probability the net profit from a drug might be negative.
This type of simulation could also be used to assess sensitivity to the parameter values (such as the judgemental probabilities) used.

4.

a) The main barrier to the use of analytical queuing theory here are the mathematically complex probability distributions. Queuing theory does, however, give us the insight that while the arrival times are very close to symmetrical around the schedule, the interview durations have a mean of 15.62 minutes, so we might conclude that ρ, the traffic intensity, is greater than 1 and so the queue of interviewees is likely to build up over time. Also not steady state.

b)

Arrivals:

Arrival relative to schedule	**Number of interviewees**	**Relative Frequency**	**Cumulative Relative Frequency**	**Random Numbers**
-2	6	.12	.12	0 - 11
-1	10	.20	.32	12 - 31
0	20	.40	.72	32 - 71
+1	8	.16	.88	72 - 87
+2	6	.12	1.00	88 - 99
Sum	50	1.00		

Interview durations:

Duration of interview (minutes)	**Number of interviews**	**Relative Frequency**	**Cumulative Relative Frequency**	**Random Numbers**
13	4	.08	.08	0 - 7
14	7	.14	.22	8 - 21
15	11	.22	.44	22 - 43
16	15	.30	.74	44 - 73
17	9	.18	.92	74 - 92
18	3	.06	.98	93 - 97
19	1	.02	1.00	98 - 99
Sum	50	1.00		

Suitable performance measures include interviewee and interviewer waiting times.

Simulation:

Interview	Scheduled Start	Interviewee Rand No.	Arrival Result	Interviewee Arrival	Interview Start	Interview Rand No.	Duration Result	Finish	Interviewee Waiting	Interviewer Waiting
1	09.00	96	+2	09.02	09.02	64	16	09.18	0	2
2	09.15	53	0	09.15	09.18	5	13	09.31	3	0
3	09.30	91	+2	09.32	09.32	3	13	09.45	0	1
4	09.45	61	0	09.45	09.45	4	13	09.58	0	1
5	10.00	21	-1	09.59	09.59	36	15	10.14	0	1
6	10.15	84	+1	10.16	10.16	63	16	10.32	0	2
7	10.30	67	0	10.30	10.32	23	15	10.47	2	0
8	10.45	49	0	10.45	10.47	90	17	11.04	2	0
9	11.00	11	-2	10.58	11.04	18	14	11.18	6	0
10	11.15	91	+2	11.17	11.18	92	18	11.36	1	0
11	11.30	32	0	11.30	11.36	76	17	11.53	6	0
12	11.45	39	0	11.45	11.53	73	16	12.09	8	0
13	12.00	15	-1	11.59	12.09	56	16	12.25	10	0
14	12.15	29	-1	12.14	12.25	40	15	12.40	11	0
Total								12.40	49	4

Calculate means and std. devs. for the performance measures:

mean interviewee waiting time = 3.5 minutes, std. dev = 4.0 minutes

mean interviewer waiting time = 0.5 minutes, std. dev = 0.8 minutes

{n.b. these waiting times are not independent}

Overrun = 10 minutes

c) Would be desirable to perform, a number of runs with different inter-interview times (e.g. 14, 15, 16 and 17) and several (at least 5?) sets of random numbers. Then take means and std. dev. of e.g. total interviewer waiting time at each inter-interview time.

Issues to discuss include number of runs, random number streams, replication, validation, assumptions, experimentation with a range of policies, statistical analysis of results, computer implementation using a spreadsheet or simulation package.

Should also discuss performance measures with the client, including issues such as the trade off between interviewer and interviewee waiting times.